NAMIBIA UNIVERSITY OF SCIENCE AND TECHNOLOGY

FACULTY OF COMPUTING AND INFORMATICS

DEPARTMENT OF COMPUTER SCIENCE

QUALIFICATION: BACHELOR OF COMPUTER SCIENCE HONOURS	
QUALIFCATION CODE: 08BCSH	LEVEL: 8
COURSE: FORMAL METHODS	COURSE CODE: FMM810S
DATE: November 2019	SESSION: 1
DURATION: 3 Hours	MARKS: 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
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This paper consists of 2 pages
(excluding this front page)

INSTRUCTIONS

1. This paper contains 6 questions.
2. Answer all questions on the exam paper.
3. Marks/scores are provided at the right end of each question
4. Do not use or bring into the examination venue books, programmable calculators, mobile devices and other materiaks that may provide you with unfair advantage. Should you be in possession of one right now, draw the attention of the examiner officer or the invigilator.
5. NUST examination rules and regulations apply.

PERMISSIBLE MATERIALS

None

Question 1 ..... [28 points]

Given a set $\mathsf{Val_R}$, consider a hierarchichal type T where each element is either of type $\mathsf{N_1}$ or $\mathsf{N_2}$. An element of type $\mathsf{N_2}$ holds a value of type $\mathsf{Val_R}$, while an element of type $\mathsf{N_1}$ holds a tuple in the following order: an element of type T, a value of type $\mathsf{Val_R}$ and an element of type T.

(a) Using type specification techniques in the Z notation, formally define T; [6]

(b) Next we introduce three functions f_1, f_2 and f_3. f_1 reverses the content of a sequence. f_2 behaves as follows when applied to an element of type T. When the element is of type $\mathsf{N_2}$, it returns a sequence containing the only element held by the type. On the other hand, when the element is of type $\mathsf{N_1}$, it returns a concatenation of the image of the first component and a sequence formed by the only element in the node, and finally concatenated with the reverse of the image of the second component (in the indicated order). Finally, f_3 returns the same node when applied to an element of type $\mathsf{N_2}$. When applied to an element of type $\mathsf{N_1}$, f_3 returns a new $\mathsf{N_1}$ tuple consisting of the image of the last component, the component in the middle and the image of the first component. Define f_1, f_2 and f_3. Note that f_2 and f_3 should apply recursively to elements of T. [18]

(c) Assuming $\mathsf{Val_R}$ is the type of natural numbers, what is the value returned by the following: [4]

$$f_2(f_3(N_1(N_2(14), 17, N_1(18)), 30, N_1(N_2(31), 32, N_2(34))))$$

You will show all steps in your calculation.

Question 2 ..... [20 points]

(a) Define a free type, DoW, that represents the days of the week from Monday to Friday [3]

(b) Consider the timetable of your university represented as a sequence S_τ, where each element is a tuple (α, β, γ) with α the day of week (of type DoW), β an alphanumeric code (of type Alpha) corresponding to the venue and γ an alphanumeric corresponding to the course code. [5]

$$\mathsf{S}_\tau = \langle (Monday, A2, DSP620S), (Tuesday, A10, SVT710S), (Wednesday, L8, SVT710S), (Monday, G50, DSP620S), (Wednesday, L2, ECN811S)\rangle$$

Using the $head$ and $tail$ operations write the expression returning the fourth element of the sequence

(c) What is the result of the following filtering: [2]

$$\mathsf{S}_\tau \upharpoonright \{d : \mathsf{DoW}, v : Alpha \bullet (d, v, SVT710S)\}$$

(d) Consider a function [10]

$$h : \mathsf{seq}\,\mathbb{N} \to \mathbb{N}$$
$$h(\langle\rangle) = 4$$
$$h(\langle \ell \rangle \frown t) = g(\ell, h(t))$$

with $g(\imath, \jmath) = 2\imath + \jmath$. What the result of $h(\langle 1, 2, 3, 4\rangle)$?

Question 3 ..... [17 points]

We wish to build a key-value store rik_{v1}. The store consists of a set of buckets, each one with its own unique identifier. Note that the identifier of a bucket is of type $Ident_r$. Inside a bucket, there is a set of function objects where the argument is a key (of type Key_r) and the imave is the value (of type Val_r) attached to it. The following operations are possible in rik_{v1}:

1. inside a bucket:
   - read the value attached to a key;
   - write or update a key attached to a value provided that neither they key nor the value exist in the bucket;
   - delete an existing key-value entry.
2. inside the store:
   - allocate five (5) buckets at a time;
   - remove an existing bucket.

Finally, note that before a bucket can be used it should be initialised. Furthermore, the overall store should be initialised.

Using the Z notation you will specify all the complex objects and the operations mentioned earlier. Your specifications will include error handling for all operations.

Question 4 ..... [15 points]

Using deduction rules in propositional and predicate logic, prove the following:

(a) $p, q \vee r \vdash (p \wedge q) \vee (p \wedge r)$ [5]

(b) $p \Rightarrow q, q \Rightarrow r \vdash p \Rightarrow r$ [5]

(c) $\neg (p \vee q) \vdash \neg p$ [5]

Question 5 ..... [5 points]

Give the truth table of the following proposition: $p \wedge (p \Rightarrow q) \wedge (q \Rightarrow r)$

Question 6 ..... [15 points]

Consider a relation R : $\{a, b, c\} \leftrightarrow \{\alpha, \beta, \gamma\}$ as follows: $\{a \mapsto \gamma, b \mapsto \gamma\}$

(a) Give in extension the following sets: dom(R) and ran(R) [2]

(b) Give in extension the following sets: $\{c\} \lhd$ R and R $\rhd \{\theta\}$ [4]

(c) Give in extension the following sets: $\{d\} \lhd$ R and R $\rhd \{\theta\}$ [4]

(d) What is the inverse of R? [2]

(e) What is the reflexive closure for R? [3]